

## Research Article

# An Analysis of Strong Coupling in the High T<sub>c</sub> Superconductors

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A new McMillan formula is assembled for the High Temperature Superconductors that evidences strong coupling in these materials. The McMillan-Hopfield parameter is calculated in each case, in addition to the Bergmann and Rainer enhancement frequency. The cubic symmetry of the CuO layers and the lattice cohesion and oxygen presence are suggested reasons for the effect.

## 1. Introduction

Since the elaboration of the microscopic theory of superconductivity by Bardeen et al. (BCS) [1], further theoretical developments have provided an explanation of how a higher T<sub>c</sub> is possible in terms of strong coupling between lattice vibrations and charge carriers. BCS theory took no account of the strong retarded time-dependant electron-phonon interaction indicated in some materials by neutron scattering, Raman, infrared, and electron spectroscopies. An approximation to the strong coupling theory formulated by McMillan [2] has so far provided an accurate correlation with data gained from high T<sub>c</sub> materials up to a possible 40°C. Beyond 40°C, however, it proved to be inaccurate and inappropriate, so it was inapplicable to the new HTSC Perovskites, discovered by Bednorz and Muller [3]. In this paper, McMillan's approximation is reformulated by using solutions of Eliashberg field equations and reassembling his formula, providing new constants in the approximate formula.

Results of band calculations to estimate the total phonon densities of states (TPDOS) and electron-phonon interaction parameters have been inappropriate for the HTSCs [4]. Estimates have been successful for non-HTSCs via (a) tunneling data [5] and (b) inelastic neutron scattering (INS) experiments that can provide accurate estimates of the TPDOS. INS data has been collected for the HTSCs [6–8] and it forms a basis for the approximation here.

Since only alloys or compounds have their T<sub>c</sub> above 10 K, finding common properties between these and the HTSCs has helped this enquiry. The A15 compounds contain

transitional elements, so do the HTSC Perovskites. Again, the A15 compounds have cubic symmetry in composition, as do the layered CuO structures in the Perovskites. A normal feature of all previously higher T<sub>c</sub> superconductors included a larger value for their electron-phonon interaction parameter  $\lambda$ . For Nb<sub>3</sub>Sn this is 1.67 and in the past a maximum has been considered to exist for this parameter above which lattice instability would occur. In this paper, it is considered that stronger rigidity is that aspect of the Perovskites structure which promotes strong coupling. Instability is therefore unlikely, and values for the interaction parameter well above 2 are shown to produce adequate grounds for the reassembly of the McMillan formula for any T<sub>c</sub> above 40 K. This is done by using solutions of the isotropic Eliashberg equations and fitting the results to the approximate formula to yield new constants. Then, it was possible to calculate the isotope shifts and the McMillan-Hopfield parameter for the Perovskite HTSCs.

## 2. Mathematical Analysis

Eliashberg equations [9] extend the BCS theory by taking into account the retarded time-dependant nature of the electron-phonon interaction due to the reduction in attenuation of the excitation's energy. Based on Migdal's theory [10] of the interaction in metals, Gorkov Green's function technique is used to treat comprehensively the damping of the excitations. With pseudopotential theory, describing the screened coulombic interaction, Eliashberg equations are confirmed in accuracy by tunneling, scattering, and critical field experiments. Their application to the HTSCs has been supported by several

authors [11–13] and must be regarded as appropriate for this purpose, given the cubic symmetry of the CuO layers in the HTSCs that would provide reduction in attenuation within the phonon spectrum. The solution of these equations in this case would require that the interaction parameter be larger than normal and this would entail that the TDOPS be much larger than that seen before in superconductors. The microscopic ingredients of Eliashberg equations include the dimensionless interaction parameter  $\lambda$  and the repulsion  $u^*$  being defined as

$$\frac{l}{u^*} = \frac{l}{u} + \frac{\omega(\text{el})\omega}{\text{ph}}. \quad (1)$$

The last term is the ratio of the electron plasma frequency propagation time to the phonon high frequency cutoff in the spectral function  $\alpha^2(\omega)F(\omega)$ . A time lag occurs in the electron pairing, reducing the electron repulsion  $u$  to an effective state  $u^*$ . The change in state to the superconducting phase then occurs at a higher temperature  $T_c$ . The interaction parameter is defined as

$$\lambda = \frac{2 \int_0^{\omega_0} \alpha^2(\omega_q) F(\omega_q) d\omega_q}{\omega_q}. \quad (2)$$

$\omega_q$  is the average phonon frequency and  $\omega_0$  is the maximum phonon frequency, easily observable from the INS data.  $\alpha^2(\omega_q)$  is the average of the electron-phonon interaction and  $F(\omega_q)$  is the phonon density of states. McMillan showed that (2) may be written as

$$\lambda = N(0) \cdot \frac{\langle I^2 \rangle}{M} \cdot \langle \omega^2 \rangle, \quad (3)$$

where  $N(0)$  is the density of electron states at the Fermi surface per single spin state,  $M$  is the mass average of the lattice atoms,  $\langle \omega^2 \rangle$  is the phonon frequency average squared, and  $\langle I^2 \rangle$  is the matrix element of the change in crystal potential  $U$  as one atom is moved. The product  $N(0) \cdot \langle I^2 \rangle$  is the McMillan-Hopfield parameter [14] designated  $n$ , representing the purely electronic contribution to the value of  $\lambda$ , while the denominator  $M \cdot \langle \omega^2 \rangle$  represents the contribution from purely phonons. Considerable discussion has arisen as to the relative contribution of each of these parameters to the value of the interaction parameter. The original McMillan equation has the form of a value for  $T_c$  the transition temperature,

$$T_c = \left( \frac{\theta_d}{l} \cdot 45 \right) \exp \left[ -\frac{1.04(1+\lambda)}{\lambda - u^* \cdot (1 + 0.6\lambda)} \right], \quad (4a)$$

$$T_c = \left( \frac{\theta_d}{A} \right) \exp \left[ -\frac{B(1+\lambda)}{\lambda - u^* \cdot (1 + C\lambda)} \right]. \quad (4b)$$

The general form is (4b) and the constants  $A$ ,  $B$ , and  $C$  are dependant upon solutions of the isotropic Eliashberg equations, to which this general form is fitted. A modified form of (4a) and (4b) derived by Dynes [15] replaces  $\theta_d$ , the Debye temperature, with  $\omega_q$ , where  $A$  becomes 1.2.

This paper determines new values for the constants  $A$ ,  $B$ , and  $C$ , accurately reflecting solutions of the field equations for any HTSC with  $T_c$  values up to about 130 K.

There is a requirement to detail the renormalization factors involved, as these are enhanced for larger interaction values. The values of  $T_c$  are determined via  $\lambda$  and  $u^*$  from the noninverted field equations because the gap parameter  $A(\omega)$  vanishes at  $T_c$ , becoming a linear equation for the infinitesimal function  $A(\omega)$ , the condition being that a nonzero solution exists. An integral equation for  $A(\omega)$  on the real frequency domain is obtained by analytical continuation of the direct matrix for  $A(i\omega_n)$  on the Matsubara imaginary frequency points  $\omega_n(2n+1)\pi T$ . McMillan, by evaluating the integral parts, obtained approximate values for the major constituents of the gap equation using a trial function  $\Delta(\omega) = \Delta_0(0 < \omega < \omega_0) = \Delta(\omega_0 < \omega)$ . The trial and computed  $\Delta(\omega)$  were to be as consistent as possible from which condition  $\Delta(0)$  and  $\Delta(\infty)$  were computed. There are now three contributions to  $\Delta(0)$ .

Consider

$$\Delta(0) = \Delta^1(0) + \Delta^2(0) + \Delta^3(0). \quad (5)$$

The first term, if thermal phonons are neglected, approximates to

$$\Delta^1(0) = \frac{\Delta_0 \lambda}{Z(0) \ln(\omega_0/T_c)}. \quad (6)$$

The second term where  $\omega_q = 0.5\omega_0$  approximates to

$$\Delta^2(0) = \frac{\Delta_\infty}{Z(0) \langle \omega \rangle \lambda / \omega_0}. \quad (7)$$

These first two terms are contributions from the electron-phonon interaction, the third being from the coulombic interaction:

$$\Delta^3(0) = - \left[ \frac{N(0)V_c}{Z(0)} \right] \left[ \Delta_0 \ln \left( \frac{\omega_0}{T_c} \right) + \Delta_\infty \ln \left( \frac{E_B}{\omega_0} \right) \right], \quad (8)$$

where  $V_c$  is the matrix element of the screened coulombic interaction averaged over the Fermi Surface and  $E_B$  is the electronic bandwidth. At high energies, the only contribution is from  $\Delta^3(0)$  and so  $Z(0) = Z(\infty)$  in (8). The renormalization  $Z$  is found to be  $Z(0) = 1 + \lambda$  and also  $Z(\infty) = 1$ . The consistency at low energies is when

$$\Delta(0) = \Delta_0 = \Delta^1(0) + \Delta^2(0) + \Delta^3(0). \quad (9)$$

And the consistency at high energies is when

$$\Delta(\infty) = \Delta_\infty = -u^* \Delta_0 \ln \left( \frac{\omega_0}{T_c} \right), \quad (10)$$

where  $u^*$  is now the coulombic pseudopotential of Morel and Anderson [16]. When (10) is now substituted into (9), the result is the strong coupling form analogous to the BSC formula for  $T_c$ :

$$T_c = \omega \cdot \exp \left[ -\frac{1 + \lambda}{\lambda - u^* \left( 1 - (\omega_q/\omega_0) \lambda \right)} \right]. \quad (11)$$

This expression must now be fitted to the solutions of Eliashberg field equations, normally done using the real frequency domain by inverse iteration. This has been done by Shiina and Nakamura [13]. Fix  $u^*$  to unity and  $T_c$  to its  $\lambda$  value for each superconducting material and assume the coupling constant in each case will remain constant over the phonon spectrum. Given the values of  $\lambda$  and the corresponding  $T_c$  values if  $\ln(\theta_d/T_c)$  versus  $(1 + \lambda)/\lambda$  is plotted, this provides a straight line with a gradient of  $B$  and an intercept of  $\ln A$ . The constant  $\omega_q/\omega_0$  is found by plotting  $(1 - A(1 + \lambda)/\ln(\theta_d/BT_c))/u^*$  against  $\lambda$  which produces a straight line of gradient of  $\omega_q/\omega_0$ . The value of  $C$  enables the value of  $\omega_q$  as  $\omega_0$  is known. Results from INS experiments on the HTSCs show that  $\omega_0$  is at least a multiple of 10 of the values for Pb and a multiple of 3 of the values for the A15 compounds.

Further work by Bergmann and Rainer [17] provides an exact matrix representation that simplifies the field equations solution. Given  $T_c$ , we find  $\lambda$  values from the expressions:

$$\begin{aligned} \det K_n = 0 &= \det (A_n + \lambda B_n), \\ K_{n,m} &= \lambda_T (n - m) + \lambda_T (m + n + 1) - 2u^* (N) \\ &\quad - \delta_{n,m} 2m + 1 + \lambda_T (0) + 2 \sum_{n=1}^m \lambda_T (1). \end{aligned} \quad (12)$$

The factor  $\lambda_T$  takes the value  $\lambda_T(0) = \lambda$  when  $n = 0$ ; a scale factor  $\pi T$  creates the standard Hermitian eigenvalue problem, where  $N$  is chosen large enough so that the largest eigenvalue is unaffected. Solutions are restricted to even. The interaction is expressed by

$$\lambda_T(n) = \frac{2 \int_0^\infty d\omega \alpha^2 F(\omega) \omega}{[2\pi n K_B T]^2} + \omega^2. \quad (13)$$

Their analysis shows that there is a phonon frequency  $2\pi T_c$  above which strong coupling starts.

An important definition is for the general moment of the average phonon frequency defined as

$$\langle \omega_n \rangle = (2\lambda) \cdot \frac{\int_0^{\omega_0} d\omega \alpha^2(\omega) F(\omega)}{\omega^{n-1}}. \quad (14)$$

If  $Q$  is the TPDOS when  $n = 1$ , then the average phonon frequency takes the form  $2Q/\lambda$ . This enables McMillan equation to be assembled in its modified form by supplying the values of  $\omega_q$ . Further  $\langle \omega^2 \rangle$  is available now, a requirement for determining the McMillan-Hopfield parameter and also  $Q$  given  $\omega_q$ .

### 3. Formulation Results

The results of Shiina and Nakamura [13] were confirmed by analytical calculation. The results are shown in Table 1. Examination of the INS data [6–8] provided values of  $\omega_q$  and  $\langle \omega^2 \rangle$  which were found by considering the PDOS for each frequency up to  $\omega_0$  and taking the mean of these. From (14), values for  $Q$  were found ranging from 18.3 meV to 81 meV.  $\lambda$  values derived from the analytical results enabled (11) to

be fitted to these results using the straight-line graph and intercept indicated previously.  $A$  was found to have a value 0.04,  $B$  a value 1.49, and  $C$  a value 0.32, thus providing a new modified McMillan formula as shown in (15). The scatter of points about the plotted line was moderate.

Consider

$$T_c = \left( \frac{\omega_q}{0.04} \right) \exp \left[ - \frac{1.49(1 + \lambda)}{\lambda - u^*(1 - 0.32\lambda)} \right]. \quad (15)$$

Clearly, the values of the constants  $A$  and  $B$  are substantially different from those in the original formula of McMillan's, reflecting the much steeper gradient in the  $T_c$  versus  $\lambda$  functional relation. The smaller value of  $C$  reflects the much larger value of the maximum phonon frequency in the HTSCs with the average being in the lower optical and higher acoustic ranges. Table 1 details the results, a value for the calculated  $T_c$ , and  $\text{Cal}T_c1$ , indicating the measure of accuracy.

A Debye form of the new formula was assembled by using  $\theta_D$  instead of  $\omega_q$ ; results are shown in Table 1.  $\text{Cal}T_c2$  is the calculated  $T_c$  using the new Debye formula for all the known HTSCs. Calculations for  $\theta_D$  have been numerous [18–21], being dependant upon the TDOPS, electronic heat capacity coefficient, and total heat capacity at constant volume. In order to minimize error, values calculated by Kulkarni et al. [18] are used for the 2T1 series. They use the same method of sample and formula preparation for three members of the series, providing an accurate basis for fitting the values of  $\theta_D$  in the materials studied. The results show that indeed very strong coupling is evidenced, since the  $T_c$  of all the HTSCs scale with their  $\theta_D$  via the new formula when the constants have the values 0.54, 1.49, and 0.32 for  $A$ ,  $B$ , and  $C$ , respectively, providing a formula:

$$T_c = \left( \frac{\theta_D}{0.54} \right) \exp \left[ - \frac{1.49(1 + \lambda)}{\lambda - u^*(1 + 0.32\lambda)} \right]. \quad (16)$$

As expected, only  $A$  changes from the modified form, reflecting the fact that  $\theta_D$  and  $\omega_q$  are intimately related. Although a  $\theta_D$  value has been established, no INS data is available for the 2T1 ( $n = 4$ ) HTSC. A value of  $\omega_q$  and  $Q$  was therefore extrapolated from the others.

The results show the strong coupling assumptions of McMillan applied to the new HTSCs and that his formula does not saturate at a  $T_c$  of 40 K, if it reassembled using the field equations for them.

### 4. Empirical Results

Following the organization of McMillan's original paper, a treatment is given for the empirical part analytically derived from the previous section. Firstly, up till now, it has been assumed that the coulombic repulsion has a value of 0.1. However, it is possible to derive accurately an estimate of this parameter from the results given here. The pseudopotential of Morel and Anderson is given by

$$u^* = \frac{N(0) V_c}{\left[ 1 + N(0) V_c \ln \left( E_B / \omega_q \right) \right]}. \quad (17)$$

TABLE 1: Results for assembling a new McMillan formula for the HTSCs.

| Material        | $\theta_D$ | Exp Tc | $\lambda$ | Q    | $\omega_q$ | CalTc1 | CalTc2 | References |
|-----------------|------------|--------|-----------|------|------------|--------|--------|------------|
| 2Tl ( $n = 3$ ) | 458        | 125    | 4.9       | 81.0 | 33.1       | 124.5  | 127.7  | [18]       |
| 2Tl ( $n = 2$ ) | 424        | 107    | 3.9       | 61.0 | 31.3       | 107.3  | 107.7  | [18]       |
| Y123            | 380        | 92     | 3.5       | 50.0 | 28.6       | 92.9   | 91.68  | [19]       |
| 2Bi ( $n = 2$ ) | 375        | 89     | 3.4       | 46.0 | 126.4      | 90.0   | 89.0   | [20]       |
| Nd ( $n = 2$ )  |            | 84     | 3.1       | 42.2 | 128.2      | 84.5   | —      | —          |
| 2Tl ( $n = 1$ ) | 370        | 80     | 2.8       | 39.2 | 28.0       | 80.7   | 78.6   | [21]       |
| La ( $n = 1$ )  | 280        | 40     | 1.7       | 18.3 | 20.9       | 40.0   | 39.9   | [20]       |

TABLE 2: Calculated isotope shifts and coulombic repulsion values.

|           | 2Tl ( $n = 3$ ) | 2Tl ( $n = 2$ ) | 2Tl ( $n = 1$ ) | Y123 | 2Bi ( $n = 2$ ) | Nd ( $n = 2$ ) | La ( $n = 1$ ) |
|-----------|-----------------|-----------------|-----------------|------|-----------------|----------------|----------------|
| $\alpha$  | 0.49            | 0.49            | 0.49            | 0.49 | 0.49            | 0.49           | 0.48           |
| $\lambda$ | 4.9             | 3.9             | 2.8             | 3.5  | 3.4             | 3.1            | 1.75           |
| $u^*$     | 0.114           | 0.105           | 0.096           | 0.1  | 0.096           | 0.093          | 0.073          |

Now, the Tc depends upon the isotopic mass of the constituent atoms directly through the Debye temperature. Using (15) and (17), we find that  $T_c \propto M^{-\alpha}$  with

$$\alpha = 0.5 \left[ 1 - \frac{(1 + \lambda)(l + 0.32\lambda)u^{*2}}{(-u^*(1 + 0.32\lambda))^2} \right], \quad (18a)$$

$$\alpha = 0.5 \left[ 1 - \left\{ u^* \frac{\ln(\theta_D/AT_c)(1 + 0.32\lambda)}{1 + \lambda} \right\} \right]. \quad (18b)$$

If the strong coupling correction is included in (18b), an exact expression for the repulsion is obtained by rearrangement in terms of  $\omega_q$  and  $\theta_D$ ; namely,

$$u^* = \frac{\text{SQR}[(1 + \lambda)(1 - 2\alpha)]}{\ln(\omega_q/0.04T_c)(l + 0.32\lambda)}. \quad (19)$$

The calculated isotope shifts and coulombic pseudopotentials are given in Table 2. A shift of 0.49 is mostly observed, though which atom this refers to is not clear, and it must be supposed that it is an average for the atoms in the CuO layers supporting conduction. The shifts for the materials originally investigated by McMillan did not show such marked similarity as those here. The BCS theory predicts an ideal value of 0.5, very near to that calculated here for the HTSCs. The calculated pseudopotential is also near to that assumed for the solution to the field equations.

McMillan also went on to provide a means of calculating  $\lambda$  given any given Tc value by rearranging his formula. If this is repeated using our new formula for the HTSCs, assuming an approximate  $u$  value of 0.1 or even an exact value as given in Table 3, a formula is provided as follows:

$$\lambda = \frac{1.49 + u^* \ln(\omega_q/0.04T_c)}{[(1 - 0.32u^*) \ln(\omega_q/0.04T_c) - 1.49]}, \quad (20)$$

from which any  $\lambda$  may be calculated.

In order to calculate the EDOS at the Fermi surface,  $N(0)$ , McMillan made use of the electronic heat capacity coefficient, this being proportional to the EDOS and an enhancement factor  $(1 + \lambda)$  from the electron-phonon interaction. Fortunately, for the HTSCs, this procedure is not necessary because the EDOS can be determined from volumetric titration, Redox titration, and Hall number. The EDOS,  $N(0)$ , is included as part of the McMillan-Hopfield parameter,  $n$  (see (3)), and some discussion has occurred concerning the relative importance of each contributing factor to the value of  $n$ . McMillan chose to view the role of  $\langle \omega^2 \rangle$  as dominant, proposing that it was merely by the softening of phonons that  $\lambda$  increases and conjecturing that the product  $N(0)\langle I^2 \rangle$  remained approximately constant in any possible series of materials. Others, notably Allen and Dynes, regarded this value as very variable and capable of increasing and thereby rendering the value of  $\lambda$  much larger. Some attempts have been made to demonstrate the existence of soft phonons in the HTSCs, but although they have been observed, no successful correlation with the onset of the superconducting effect has been demonstrated in the HTSCs.

Values of ingredients and the calculated McMillan-Hopfield parameter  $n$  eV/A are shown in Table 3. Included are the unit cell mass average  $M$ , the mean square of the phonon frequencies  $\langle \omega^2 \rangle$  meV, and the Bergmann and Rainer enhancement temperature  $2\pi T_c$  meV. For comparison, values for  $\text{Nb}_3\text{Sn}$  and Pb are included.

The values of  $\langle \omega^2 \rangle$  and  $n$  clearly increase as Tc and  $\lambda$  increase, and if the values of  $M$  are expressed in MeV, the relation expressed by (3) is held accurately. These large values of  $n$  suggest that either the EDOS at the Fermi surface is larger for any HTSC Tc values, or the change in crystal potential as lattice vibrations occur becomes very large. Since it is known that the EDOS is not large for the HTSCs at the Fermi surface, we may conclude that the crystal potential changes



TABLE 3: HTSC data for the McMillan-Hopfield parameter.

|                    | ExTc | $2\pi T_c$ | $M$  | $\langle\omega^2\rangle$ | $n$  | $\lambda$ |
|--------------------|------|------------|------|--------------------------|------|-----------|
| 2Tl ( $n = 3$ )    | 125  | 67.6       | 59.6 | 1253.2                   | 49.3 | 4.9       |
| 2Tl ( $n = 2$ )    | 107  | 57.9       | 65.2 | 1122.3                   | 38.4 | 3.9       |
| 2Tl ( $n = 1$ )    | 80   | 43.3       | 76.6 | 900.0                    | 25.9 | 2.8       |
| Y123               | 92   | 49.8       | 51.2 | 936.4                    | 22.6 | 3.5       |
| 2Bi ( $n = 2$ )    | 89   | 48.2       | 59.2 | 795.4                    | 21.6 | 3.4       |
| Nd ( $n = 2$ )     | 84   | 45.5       | 53.5 | 912                      | 20.3 | 3.1       |
| La ( $n = 1$ )     | 40   | 21.7       | 56.8 | 501.8                    | 6.7  | 1.75      |
| Nb <sub>3</sub> Sn | 18   | 9.75       | 99.3 | 357.2                    | 7.9  | 1.67      |
| Pb                 | 7.2  | 3.9        | 207  | 31.4                     | 2.4  | 1.55      |

dramatically, probably due to covalent bonding of the oxygen atoms in the CuO layers.

## 5. Discussion

As a summary, solutions of the isotropic Eliashberg equations have yielded appropriate interaction parameter values for each HTSC material. The time of flight INS experiments provided values for the average phonon frequencies,  $\omega_q$ , and phonon frequencies squared average  $\langle\omega^2\rangle$ , from which the TPDOS was calculated (this can be done from inspecting the INS data directly). Having the appropriate Debye temperature values, it was possible to assemble a new McMillan formula for the HTSC series of Perovskites. This enables accurate deductions of the isotope shifts, the Coulombic repulsion  $u^{*f}$ , and the McMillan-Hopfield parameter  $n$ , having calculated the unit cell mass average in each case.

In detailing the reasons for the very high transition temperatures for the HTSC Perovskites, it must now seem correct, given the evidence, to retain involvement with the now well-established theory of strong coupling and the physical reasons for it, in terms of crystal lattice-mobile charge carrier interaction. A general description of this is that a phonon-induced interaction occurs between carrier one and lattice at a time  $(x, t)$  and with certain coupling constant  $\lambda$  creating a disturbance that may be Fourier transformed as a phonon. It propagates several orders of magnitude more slowly than the carrier velocity at the Fermi surface  $(v, t)$ . In order to interact, the second carrier must wait for the first to move away and for the lattice disturbance to develop exactly by  $\pi$  radians, doing so at time  $(x', t')$  with an assumed coupling constant equal to that of the first carrier interaction. The second carrier has received the phonon then transfers it back to the first, with a consequent effect on the pair correlation and a reduction in Coulombic repulsion between them. These Cooper pairs then have a coherence length according to Ginzburg-Landau theory, given by  $SQR [h^2/2m|a|]$ .

The carriers are however not the simple entities of the Bloch-Sommerfeld model but are quasiparticles, clothed in their interaction with the lattice and themselves and they are therefore renormalized, being slowed down as a consequence. If the TDOPS is high then so will be the renormalization. McMillan's approximation theory shows that the carrier velocity at the Fermi surface is renormalized by a factor

$(1 + \lambda)$ . Thus, in order for the second carrier to receive the disturbance within just  $\pi$  radians of the phonon's cycle in a material with high renormalization, it must be closer to the first carrier than usual. It is known that the coherence length for the HTSCs is unusually small, ranging from 0.5 nm to 2.0 nm. The cyclotron masses and electron specific heats are both renormalized by a factor  $1/(1 + \lambda)$ , and for this reason the interaction parameter is the most influential parameter for the system.

There exist several reasons why there should be a much higher TDOPS in the Perovskites. Firstly, the presence of transitional elements would greatly enhance lattice rigidity. The reason is that they contain  $d$  states, comparable in energy to their valency states in the  $s$  band. These localized band  $d$  states extend far from the nucleus requiring a compensatory potential well on the atom that contracts the  $s$  states towards the nucleus. The nearest neighbor atomic spacing is reduced and as a consequence the lattice rigidity and structural stability is enhanced.

Further to this, the cubic symmetry of the CuO layers must, as a consequence of being an ordered system, reduce attenuation of lattice phonons, order promotes, and communicates order. Both of these circumstances lead inevitably to strong coupling theory. A detailed analysis of electrostatic cohesion in the Perovskites may be found in Harrison's book [22], which is difficult to improve upon.

In addition to renormalization, strong coupling theorists established that phonon damping by lattice attenuation above the average phonon frequency for normal superconductors was not negligible, as the BSC theory had assumed. Indeed, low frequencies in the acoustic range of low energy make superconductivity impossible except at temperatures near to absolute zero. Anderson and Morel considered that the phonon-induced interaction between lattice and charge carriers is mediated primarily through short wavelength phonons. While Bergmann and Rainer showed that, more exactly, there is an enhancement at a frequency of  $2\pi T_c$  MeV, as shown in Table 3 for the HTSCs. Damping is due to several causes: thermal phonons from the heat content of the lattice generating random vibrations, simple metallic disorder, dislocations at grain boundaries or impurities, and interstitial charge carriers moving perpendicular to the direction of phonon propagation. McMillan specifically neglected thermal phonons, while Eliashberg calculated the attenuation

effect of thermal phonons near to and far from the Fermi surface. As a result of these studies, strong coupling indicates that more phonons in the higher frequency range exist than had hitherto been thought important. Now, inspection of the INS data for the HTSCs shows that, as  $T_c$  increases, the density of phonons above the average and near to  $2\pi T_c$ , the Bergmann and Rainer enhancement frequency, increases. Further inspection of the INS data shows that the maximum frequency is many times that of  $\text{Nb}_3\text{Sn}$ . Eliashberg's analysis shows that the maximum frequency is the dominant parameter affecting the carrier excitation spectrum.

Just as the McMillan-Hopfield parameter is dominated by the phonon frequency square average, the damping reduction is dominated by the maximum frequency square average to the tune of two orders of magnitude larger than normal superconductors. The scenario is exemplified by the  $\text{Hg}(1223)$  superconductor, where pressures of 160 kbar induce artificially high critical temperatures of around 153 K. An explanation of this would be that such pressures reduce damping by reducing thermal phonons. The increase in structural stability as a result of this pressure corresponds to the effect in the normal HTSCs, of CuO layer lattice symmetry, and the presence of the transitional elements increasing nearest neighbor spacing resulting in increased stability.

The reason why very large maximum phonon frequencies occur in the HTSCs could be sought in a larger than normal oxygen content situated in the CuO layers. The oxygen is lighter and vibrates at a higher frequency than the other constituent atomic species. For this reason, the generalised phonon density of states increases by a factor of 2.5 due to the presence of oxygen; for most metals, it is considerably lower, including the metals Tl, Ba, Ca, and Cu having similar weighting factor values.

## 6. Conclusions

A number of different models have been proposed to explain the phenomenon of the new HTSCs, soft phonons, Van Hove singularities, magnetic pairing, the appearance of Bipolarons, and several others. However, why we should not feel constrained by existing theory has never been demonstrated to any degree of certainty. Our first main duty is to examine what similarities exist between existing higher  $T_c$  superconductors and the new breed and also find valid reasons for rejecting strong coupling. This has been clearly pointed out here, that the CuO layers have a cubic symmetry. They have neighboring transitional elements that pack nearest neighbors into a more rigid lattice than normal. The existence of a coherent, phonon-induced state that reduces coulombic repulsion between charge carriers is evidenced everywhere. The strong coupling theory of Migdal and Eliashberg is the first theory to consider, and the only one to fit the data obtained is proposed here. The idea of lattice instability advanced as a reason for rejecting strong coupling theory clearly did not take account of the fact that enhanced stability and rigidity were a requirement for this coupling to take place. The very large TDOPS arises because stability and symmetry reduce attenuation in the high frequency phonons near to the Bergmann and Rainer

enhancement frequency that arises due to the presence of lighter oxygen atoms. The fact that  $T_c$  increases every time a CuO layer adds to the next material in a series is crucial to the arguments here. It is not disputed here that other phenomena occur in the scenario but that they are neither necessary nor sufficient conditions for superconductivity. Using angle resolved photoemission spectroscopy, Shen and coworkers argue consistently for strong coupling theory. Some workers [23] have provided experimental data supporting the fact that antiferromagnetic charge carrier spin alignment is a counterpart to superconductivity in some HTSCs. This would not be surprising since the coherence length is so small and ordering already exists, so that exchange forces could come into play.

## Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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